£1,000, and that a bonus of £1. 10*s*. per cent. per annum is added thereto in the usual manner each five years. To avoid any invidious distinction, I have purposely chosen data that, as far as I know, do not accurately represent the operations of any Office.

The first column shows the (net) annual premium required to assure £1,000 at the ages stated; the second, the annual premium required to meet the increasing benefit above described; and the third, the ratio of the first to the second.

Age.	Premium for £1,000.	Premium for £1,000, with Bonus.	Ratio.
20	14·9358	24·3651	1·63
30	19·5192	29·6399	1·52
40	25·9932	36·8245	1·42
50	36·2236	47·7889	1·32
60	57·8955	70·1644	1·21

Assuming the Office to charge the premium in the first column with the ordinary "loading" of 30 per cent., it will be seen that, while the entrants at age 50 get a reasonable equivalent for their payments, those at the more advanced ages have to pay some 30 per cent. of the premiums of their more fortunate younger brethren of 20.

I remain, Sir,

Your most obedient servant,

Aberdeen, 1st February, 1859. H. A. S.

ON THE FACILITIES AFFORDED IN THE COMMUTATION SYSTEM BY THE INTRODUCTION OF COLUMNS OF DIFFERENCES.

To the Editor of the Assurance Magazine.

SIR,—Although your correspondent, "Joshua Milne," will not admit the superiority of the columnar method in life contingency calculations, he has failed, I think, to prove that, even with the help of a complete set of temporary and deferred annuities, the " ancient" method gives facilities equal to those which are claimed for its rival. I regret that the examples which have been adduced, in illustration of the two methods, were not presented by the writers on both sides, with the operations in full, for the merits of each method could then have been better appreciated.

As the old method of computation is, doubtless, greatly facilitated by the tabulation of the annuities, temporary and deferred, so may the power of the columnar method be increased by the tabulation of the differences of the N and M columns, and the summation of those differences, to be used supplementally to the D and N table in its ordinary form. I have long been sensible of the importance of such tables (and doubtless it has been equally apparent to others), for, as far back as the early part of 1854, I tabulated the differences of the N and M columns for terms from 1 to 70 years, and from those differences formed new R and S columns.

As a discussion has recently taken place in your valuable *Journal* with regard to the relative merits of the old and the columnar methods, it may not be uninteresting to give a short account of the difference tables above

referred to; I therefore propose, if you deem it worthy of insertion, to lay before your readers a short description of the mode of computing them adopted by me, as well as a few formulæ illustrative of the advantages to be derived from such a supplement to the original table.

The tables consist of a series of columns, supplemental to the columns N, S, M, R, of the table in its ordinary form, and are calculated for every age, from 0 to 70 (*i. e.* commencing with 0—1 and terminating with 69—70), which is deemed sufficiently extensive for practical purposes.

At the time of computing them, I was not, nor am I at present, aware of any suggestion having been made for a similar undertaking, and am therefore entitled to claim originality, both for the construction of the tables, and for the adaptation of the common formulæ to them.

The table used as the basis of these supplemental tables is the English Life Table, No. 1, contained in the *Appendix to the Sixth Annual Report of the Registrar-General*, 1844, pp. 604-5 (8vo. edition).

With regard to the arrangement of the tables, it is necessary to state, that the one adopted by Dr. Farr, as regards columns N and S, has been followed;—N_x and S_x corresponding with N_{x-1} and S_{x-1} of Jones, Davies, and others; and the following relation will be found to subsist between the several columns, viz.:—

In the ordinary table.

$N_x = D_x + D_{x+1} + \ldots D_{x+z}$; z being the oldest age in the table.
$S_x = N_x + N_{x+1} + \ldots N_{x+z}$; „ „
$M_x = C_x + C_{x+1} + \ldots C_{x+z}$; „ „
$R_x = M_x + M_{x+1} + \ldots M_{x+z}$; „ „

In the supplemental tables.

$N_{x|n} = D_x + D_{x+1} + \ldots D_{x+n-1}$; n being the age at which the risk will cease, and at which the table also terminates.

$S_{x|n} = N_{x|n} + N_{x+1|n} + \ldots N_{x+(n-1)|n}$; „ „
$M_{x|n} = C_x + C_{x+1} \ldots\ldots C_{x+n-1}$; „ „
$R_{x|n} = M_{x|n} + M_{x+1|n} \ldots M_{x+(n-1)|n}$; „ „

The symbols $N_{x|n}$ and $M_{x|n}$ are used to express not simply an abridgment of the terms $N_x - N_{x+n}$ and $M_x - M_{x+n}$, as adopted by Professor De Morgan, Dr. Farr, and others, but rather as the index to the tabulated results of their differences. The symbols $S_{x|n}$ and $R_{x|n}$* in like manner indicate the summation of the columns $N_{x|n}$ and $M_{x|n}$, and are equivalent to the result of the several terms $S_x - S_{x+n} - nN_{x+n}$ and $R_x - R_{x+n} - nM_{x+n}$, and must not be confounded with the abbreviated form of $S_x - S_{x-n}$ and $R_x - R_{x+n}$.

The columns of the supplemental tables bear the following relation to the corresponding columns of the original table:—

$$N_{x|n} = N_x - N_{x+n}.$$
$$S_{x|n} = S_x - S_{x+n} - n\,.\,N_{x+n}.$$
$$M_{x|n} = M_x - M_{x+n}.$$
$$R_{x|n} = R_x - R_{x+n} - n\,.\,M_{x+n}.$$

and were computed in the manner now to be explained.

* Professor De Morgan adopted the symbols $S_{x|n}$ and $R_{x|n}$ as an abbreviated form for $S_x - S_{x+n} - nN_{x+n}$ and $R_x - R_{x+n} - nM_{x+n}$.—*Companion to Almanac*, 1840, p. 8.

$$\text{Since } N_x - N_{x+1} = D_x,$$

Table I., terminating at age 1, column $N_{x|n} = N_{0|1}$, was found by subtracting N_1 from $N_0 = D_0$, which was then inserted against age 0.

$$\text{And since } N_x - N_{x+n} = D_x + D_{x+1} + D_{x+2} + \ldots D_{x+n-1};$$
$$\text{and } N_x - N_{x+1+n} = D_x + D_{x+1} + D_{x+2} \ldots D_{x+n}, \text{ \&c.}$$

Table II., terminating at age 2, was formed by adding to the number opposite age 0 of the original table the quantity D_1; and the last tabulated age, being one year less than that at which the table terminates, $N_{1|2} = D_1$.

In like manner, the subsequent tables were constructed, by adding to the numbers opposite each age respectively in the table terminating at an age one year younger than the final age of the table in course of formation, the constant quantity D_{x+n-1}; $x+n$ representing the age at which the latter table terminates.

An example of the process used in constructing column $N_{x|n}$ is here subjoined:—

$N_{0\vert 1} = D_0 =$	51274·00						
$D_1 =$	41848·13	$N_{0\vert 2}$	93122·13				
		D_2	38069·94	$N_{0\vert 3}$	131192·07		
		$N_{1\vert 2}$	41848·13	D_3	35706·85	$N_{0\vert 4}$	166898·92
		D_2	38069·94	$N_{1\vert 3}$	79918·07	D_4	33819·67
				D_3	35706·85	$N_{1\vert 4}$	115624·92
				$N_{2\vert 3}$	38069·94	D_4	33819·67
				D_3	35706·85	$N_{2\vert 4}$	73776·79
						D_4	33819·67
						$N_{3\vert 4}$	35706·85
						D_4	33819·67
	*	*	*		*	*	
	*	*	*		*	*	

$N_{0\vert 67}$	968510·768						
D_{67}	1950·862	$N_{0\vert 68}$	970461·630				
$N_{1\vert 67}$	917236·768	D_{68}	1792·741	$N_{0\vert 69}$	972254·371		
D_{67}	1950·862	$N_{1\vert 68}$	919187·630	D_{69}	1640·158	$N_{0\vert 70}$	973894·529
$N_{2\vert 67}$	875388·638	D_{68}	1792·741	$N_{1\vert 69}$	920980·371		
D_{67}	1950·862	$N_{2\vert 68}$	877339·500	D_{69}	1640·158	$N_{1\vert 70}$	922620·529
$N_{3\vert 67}$	837318·698	D_{68}	1792·741	$N_{2\vert 69}$	879132·241		
D_{67}	1950·862	$N_{3\vert 68}$	839269·560	D_{69}	1640·158	$N_{2\vert 70}$	880772·399
$N_{4\vert 67}$	801611·848	D_{68}	1792·741	$N_{3\vert 69}$	841062·301		
D_{67}	1950·862	$N_{4\vert 68}$	803562·710	D_{69}	1640·158	$N_{3\vert 70}$	842702·459
$N_{5\vert 67}$	767792·178	D_{68}	1792·741	$N_{4\vert 69}$	805355·451		
D_{67}	1950·862	$N_{5\vert 68}$	769743·040	D_{69}	1640·158	$N_{4\vert 70}$	806995·609
*	*	D_{68}	1792·741	$N_{5\vert 69}$	771535·781		
*	*	*	*	D_{69}	1640·158	$N_{5\vert 70}$	773175·939
$N_{64\vert 67}$	6852·673	*	*	*	*		
D_{67}	1950·862	$N_{64\vert 68}$	8803·535	*	*	*	*
$N_{65\vert 67}$	4396·880	D_{68}	1792·741	$N_{64\vert 69}$	10596·276	*	*
D_{67}	1950·862	$N_{65\vert 68}$	6347·742	D_{69}	1640·158	$N_{64\vert 70}$	12236·434
$N_{66\vert 67}$	2114·245	D_{68}	1792·741	$N_{65\vert 69}$	8140·483		
D_{67}	1950·862	$N_{66\vert 68}$	4065·137	D_{69}	1640·158	$N_{65\vert 70}$	9780·641
		D_{68}	1792·741	$N_{66\vert 69}$	5857·848		
		$N_{67\vert 68}$	1950·862	D_{69}	1640·158	$N_{66\vert 70}$	7498·006
		D_{68}	1792·741	$N_{67\vert 69}$	3743·603		
				D_{69}	1640·158	$N_{67\vert 70}$	5383·761
				$N_{68\vert 69}$	1792·741		
				D_{69}	1640·158	$N_{68\vert 70}$	3432·899
						$N_{69\vert 70}$	1640·158

In practice, the above constants, distinguished by the antique type, would

be set down on a card and added to each line of the preceding column, by which means the several columns may be produced with great rapidity.

Verification of the work was obtained as follows:—

$$N_{0|n}=N_0 - N_{0+n}$$
$$N_{1|n}=N_{0|n}-D_0$$
$$N_{2|n}=N_{0|n}-D_0-D_1=N_{1|n}-D_1$$
$$\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot$$
$$N_{x|n}=N_{0|n}-D_0-D_1-D_2 \ldots D_{n-1}=N_{x-1|n}-D_{n-1}.$$

Further verification of columns $N_{x|n}$ was effected by the additions of columns $S_{x|n}$, the quantities of which being $N_{x|n}+N_{x+1|n}+N_{x+2n}+\ldots+N_{x+n-1|n}$, and agreeing with $S_x-S_{x+n}-nN_{x+n}$ of Table No. 1, further proved their accuracy.

Column $S_{x|n}$ was formed by addition, in the usual manner, and age 0 was checked by $S_x-S_{x+n}-nN_{x+n}$, and further verified by $S_{x|n}-N_{x|n}=S_{x+1|n}$.

Column $M_{x|n}$ was formed from column C, in the same manner as column $N_{x|n}$ from column D, and column $R_{x|n}$ by the same process as column $S_{x|n}$.

By the aid of the supplemental tables, the computer will find that in the cases for which they are especially intended,—viz., for finding values for periods short of the whole duration of life, all the terms in the usual formulæ required to reduce the value of a whole life assurance or annuity, whether for fixed or increasing sums, to the value of a temporary assurance or annuity, are rendered unnecessary: in other words, the expression for whole life risks will apply equally to temporary risks, by substituting $x|n$ for x, $x+n|n$ for $x+n$, &c.

Take, for example, one of the most simple cases:—the formula for the present value of a whole life annuity is $\frac{N_{x+1}}{D_x}$; for that of a temporary annuity it is $\frac{N_{x+1}-N_{x+1+n}}{D_x}$, which, by the supplemental tables, becomes $\frac{N_{x+1|n}}{D_x}$: thus saving one tabular entry and a subtraction, the importance of which, in forming a series of values, cannot fail to be appreciated.

In calculations in which both numerator and denominator are of a temporary character, a greater advantage will be gained. The common formula for the annual premium for a deferred temporary annuity is $\frac{N_{x+1+n}-N_{x+1+n+n'}}{N_x-N_{x+n}}$; but by these tables it becomes $\frac{N_{x+1+n|n'}}{N_{x|n}}$. By the former, four tabular entries and two subtractions are required; while by the latter, two tabular entries only produce the same result.

The facilities afforded by this new arrangement will, however, be more apparent from an example involving more elements in the calculation than the preceding cases. For a deferred annuity with return of premiums, the common expression is $\frac{N_{x+1+n}}{N_x+R_{x+n}+nM_{x+n}-N_{x+n}-R_x}$; but by the supplemental tables it becomes $\frac{N_{x+1+n}}{N_{x|n}-R_{x|n}}$, agreeing, *mutatis mutandis*, with the formula for the annual premium for assuring a sum at death with return of all the premiums—viz., $\frac{M_x}{N_x-R_x}$.

It will therefore, I think, be apparent that columns $N_{x|n}$ and $M_{x|n}$ —giving, as they do, by inspection, the differences of the numbers opposite every present and increased age, from 0 to 70, of columns N and M of the table in its common form; and that columns $S_{x|n}$ and $R_{x|n}$, containing the summation of columns $N_{x|n}$ and $M_{x|n}$, in the same manner as columns S and R are the summation of columns N and M; and thus sustaining the same relation to each other in temporary cases as do columns N, S, M, R, of the ordinary table for cases involving the whole of life—afford the means of calculating assurances and annuities for terms, as concisely, and with as much facility, as those for the whole period of existence.

In conclusion, I annex a few formulæ adapted both to the ordinary and the supplemental tables, which will suffice to show their relative value, and indicate the great saving of labour effected by tables computed as here pointed out, over those in common use. There are, however, I am aware, a few cases in which no especial advantage is gained by the use of the new tables; but although, in such cases, they are not serviceable to the same extent as in others, they will, nevertheless, be found useful for independent verification of computations by the common formulæ.

FORMULÆ.

Annual premium to assure £1 at death, if within n years :—

By ordinary table.	*By supplemental tables.*
$\dfrac{M_x - M_{x+n}}{N_x - N_{x+n}}.$	$\dfrac{M_{x\|n}}{N_{x\|n}}.$

Annual premium to assure £1 at death if between the ages of $x+n$ and $x+n+n'$:—

$\dfrac{M_{x+n} - M_{x+n+n'}}{N_x - N_{x+n+n'}}.$	$\dfrac{M_{x+n\|n'}}{N_{x\|n+n'}}.$

Annual premium for an assurance for n years, commencing at £A and increasing £P per annum:—

$\dfrac{(A-P)(M_x - M_{x+n}) + P(R_x - R_{x+n} - n.M_{x+n})}{N_x - N_{x+n}}.$	$\dfrac{(A-P)M_{x\|n} + P.R_{x+n}}{N_{x\|n}}.$

The same when P=A:—

$\dfrac{A(R_x - R_{x+n} - M_{x+n})}{N_x - N_{x+n}}.$	$\dfrac{A.R_{x\|n}}{N_{x\|n}}.$

Annual premium for an assurance of £A for $n+1$ years; after which it increases by £P for $n-1$ years, and then stops:—

$\dfrac{(A-P)(M_x - M_{x+n+n'}) + P.(R_{x+n} - R_{x+n+n'} - n'.M_{x+n+n'})}{N_x - N_{x+n+n'}}.$	$\dfrac{(A-P)M_{x\|n+n'} + P.R_{x+n\|n'}}{N_{x\|n+n'}}.$

Temporary premium—£p now and $n-1$ times more, n times in all—for £1 at death:—

$\dfrac{M_x}{N_x - N_{x+n}}.$	$\dfrac{M_x}{N_{x\|n}}.$

Temporary premium, increasing by a proportion, to last n years only, last premium $\{1+(n-1)\pi\}p$, for £1 at death:—

$$\frac{\mathrm{M}_x}{\mathrm{N}_x-\mathrm{N}_{x+n}+\pi(\mathrm{S}_{x+1}-\mathrm{S}_{x+n-1}-n\,\mathrm{N}_{x+n})};$$

or, $$\frac{\mathrm{M}_x}{(1-\pi)(\mathrm{N}_x-\mathrm{N}_{x+n})+\pi(\mathrm{S}_x-\mathrm{S}_{x+n}-n\,\mathrm{N}_{x+n})}.$$

$$\frac{\mathrm{M}_x}{(1-\pi)\mathrm{N}_{x|n}+\pi\,\mathrm{S}_{x|n}}.$$

Annual premium for endowment assurance of £1 at end of n years or at previous decease; n payments:—

$$\frac{\mathrm{D}_{x+n}+\mathrm{M}_x-\mathrm{M}_{x+n}}{\mathrm{N}_x-\mathrm{N}_{x+n}}.$$

$$\frac{\mathrm{D}_{x+n}+\mathrm{M}_{x|n}}{\mathrm{N}_{x|n}}.$$

Annual premium for an endowment payable at age $x+n$, with return of all the premiums if death take place previously; n payments in all:—

$$\frac{\mathrm{D}_{x+n}}{\mathrm{N}_x+\mathrm{R}_{x+n}+n\,\mathrm{M}_{x+n}-\mathrm{N}_{x+n}-\mathrm{R}_x}.$$

$$\frac{\mathrm{D}_{x+n}}{\mathrm{N}_{x|n}+\mathrm{R}_{x|n}}.$$

Extract from the Tables referred to. TERMINATING AT AGE 60.

Present Age.	$\mathrm{N}_{x\|n}$.	$\mathrm{S}_{x\|n}$.	$\mathrm{M}_{x\|n}$.	$\mathrm{R}_{x\|n}$.	Present Age.
0	950014·739	18321624·119	20411·3336	224834·5381	0
1	898740·739	17371609·380	12479·2936	204423·2045	1
2	856892·609	16472868·641	9919·2026	191943·9109	2
3	818822·669	15615976·032	8665·4586	182024·7083	3
4	783115·819	14797153·363	7817·8420	173359·2497	4
5	749296·149	14014037·544	7232·1306	165541·4077	5
6	717047·909	13264741·395	6778·2141	158309·2771	6
7	686192·119	12547693·486	6426·9586	151531·0630	7
8	656586·469	11861501·367	6153·8230	145104·1044	8
9	628116·589	11204914·898	5940·7591	138950·2814	9
10	600688·489	10576798·309	5774·8262	133009·5223	10
11	574225·449	9976109·820	5645·5128	127234·6961	11
12	548662·859	9401884·371	5519·9659	121589·1833	12
13	523970·069	8853221·512	5398·7566	116069·2174	13
14	500117·489	8329251·443	5254·6329	110670·4608	14
15	477104·159	7829133·954	5103·1535	105415·8279	15
16	454912·329	7352029·795	4952·9703	100312·6744	16
17	433517·029	6897117·466	4804·7413	95359·7041	17
18	412893·399	6463600·437	4657·8927	90554·9628	18
19	393017·309	6050707·038	4512·4698	85897·0701	19
20	373865·319	5657689·729	4369·0678	81384·6003	20
21	355414·659	5283824·410	4227·1548	77015·5325	21
22	337643·219	4928409·751	4087·2876	72788·3777	22
23	320529·519	4590766·532	3948·4541	68701·0901	23
24	304052·709	4270237·013	3812·1885	64752·6360	24
25	288192·529	3966184·304	3677·0261	60940·4475	25
26	272929·299	3677991·775	3543·4820	57263·4214	26
27	258243·909	3405062·476	3412·0268	53719·9394	27
28	244117·809	3146818·567	3282·2150	50307·9126	28
29	230532·969	2902700 758	3154·0624	47025·6976	29
30	217471·879	2672167·789	3027·9945	43871·6352	30
31	204917·539	2454695·910	2903·1986	40843·6407	31
32	192853·429	2249778·371	2780·4841	37940·4421	32
33	181263·509	2056924·942	2659·4587	35159·9580	33
34	170132·189	1875661·433	2540·1280	32500·4993	34
35	159444·349	1705529·244	2422·8515	29960·3713	35
36	149185·273	1546084·895	2306·9206	27537·5198	36
37	139340·684	1396899·622	2193·0264	25230·5992	37

Extract from the Tables referred to—(continued).

Present Age.	$N_{x\|n}$.	$S_{x\|n}$.	$M_{x\|n}$.	$R_{x\|n}$.	Present Age.
38	129896·709	1257558·938	2081·1486	23037·5728	38
39	120839·872	1127662·229	1970·9507	20956·4242	39
40	112157·084	1006822·357	1862·4296	18985·4735	40
41	103835·627	894665·273	1755·8788	17123·0439	41
42	95863·148	790829·646	1650·9866	15367·1651	42
43	88227·647	694966·498	1548·0273	13716·1785	43
44	80917·467	606738·851	1446·9774	12168·1512	44
45	73921·280	525821·384	1347·8129	10721·1738	45
46	67228·087	451900·104	1250·5098	9373·3609	46
47	60827·201	384672·017	1155·0437	8122·8511	47
48	54708·244	323844·816	1061·3902	6967·8074	48
49	48861·128	269136·572	969·5246	5906·4172	49
50	43276·065	220275·444	879·6504	4936·8926	50
51	37943·540	176999·379	791·7295	4057·2422	51
52	32854·316	139055·839	705·7244	3265·5127	52
53	27999·422	106201·523	621·3893	2559·7883	53
54	23370·146	78202·101	539·1052	1938·3990	54
55	18958·030	54831·955	458·6274	1399·2938	55
56	14754·862	35873·925	372·6612	940·6664	56
57	10760·094	21119·063	282·1509	568·0052	57
58	6972·152	10358·969	189·9551	285·8543	58
59	3386·817	3386·817	95·8992	95·8992	59

I am, Sir,

Your obedient servant,

SAMUEL L. LAUNDY.

Eagle Life Office,
1st March, 1859.

NEW GERMAN WORKS ON LIFE ASSURANCE.

To the Editor of the Assurance Magazine.

SIR,—I wish to direct your attention to two new German publications referring to life assurance—1. *Die Lebensversicherungs Praxis* (the Practice of Life Assurance), by Dr. August Wiegand, the able Director of the Life Insurance Society "Iduna," at Halle. This book is principally addressed to life insurance agents, to explain to them the means they should take to extend the practice of life assurance, which, you know, is in Germany still used on a very limited scale. Dr. Wiegand is renowned for his capacity in treating scientific questions in a popular way, and there can be no doubt this little work will do a great deal of good. It consists of two parts. The first explains the essence and the importance of life insurance in general; it enumerates the different kinds of life assurance, and then points out to persons of different ranks of life the peculiar advantages they may derive therefrom: the second part teaches the agent how to manage, and gives him a complete plan to overcome the difficulties and prejudices which oppose themselves to his activity. You may best judge about the book if I add the contents, according to the index, and if I state that all the chapters are excellently executed:—1, the first beginning of life assurance—the Sterbecassen (a kind of friendly Societies); 2, on what principles they are generally established; 3, their objects; 4, life insurance, single life; 5, for a short duration;